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Assessment on the Fair valuation of the S&P 500

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Introduction

This report details the valuation method used in the estimation of the fair value of the S&P 500 index. The valuation technique of choice is the Discounted Dividend Model (DDM), which enables us to value the index by utilising the aggregate dividend from the S&P 500. The report further details a comprehensive explanation of the assumptions and processes used in valuing the index using the DDM, its strengths and weaknesses, and a sensitivity analysis of the index value to changes in assumptions. We also perform sanity checks using several models listed in the appendix, and outline our results.

Background

The S&P 500 index is a stock market index which tracks 500 leading American companies in various number of industries. The S&P 500 is market capitalization-weighted, which means that the firms with larger market capitalization have a greater impact on the index's value compared to those firms with a smaller market capitalization. The index is calculated using the formula below:

$$Index\ Value = \frac{\sum(P_i \times Q_i)}{Divisor}$$

Where P_i represents the share price of a company i and Q_i represents the outstanding shares of the same company i. Essentially, this equation could also be interpreted as the sum of the market capitalization of all the companies included in the index, divided by a particular divisor.

The Divisor enables the index value to be scaled down to an easily representable number instead of dealing directly with the dollar value of the total market capitalization of the

companies in the index. The Divisor also plays a crucial role in ensuring that the index value is not altered by events such as corporate actions which might influence the value of a company's stock. If a corporate action causes a stock's value to change, necessary adjustments to the divisor are made to offset this change.

The S&P 500 is widely used in the financial industry today, as it is seen as the best way to track the performance of U.S stock market due to the diverse nature of companies included in the index.

The Dividend Discount Model

The dividend discount model (DDM) is a model used for valuing equity, which entails finding the present value of the expected dividends of a particular stock. The dividend discount model assumes that the shareholder's cash flows are represented in the form of dividend payments, and the sale of the stock under analysis. The model is represented using the equation(1) below:

$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \ldots + \frac{Div_N}{(1+r_E)^N} + \frac{P_N}{(1+r_E)^N}$$

Where P_0 is the present value of the stock under analysis. $Div_{1..N}$ represent the dividends paid to the shareholder at the end of years 1 up to N. P_N is the future price of the stock given that the shareholder expects to sell the stock in the future, and r_E is the equity cost of capital. Due to the nature of the risky cash flows, they cannot be discounted using the risk-free interest rate. They are discounted using the equity cost of capital, which is the shareholder's expected rate of return on the investment.

In using the DDM, we can make a few assumptions:

1. We can assume that the dividends paid to the shareholder remains constant perpetually,
2. We can assume that the dividends paid to the shareholder grow at a constant rate perpetually
3. We can assume that the dividends paid to the shareholders are forecasted for some years into the future (e.g 3 years), thereafter, we then make the assumption that the dividend stream grows at a constant rate, or remains constant

Reasons for using DDM

The DDM is useful for companies with stable growth rate and with the growth rates of S&P 500 staying relatively stable, it seemed a reasonable choice. It is also includes relatively simple, straightforward and logically correct calculations (it relies on only a few parameters) so doesn't require too much technical expertise to evaluate.

Estimating Fair Value of S&P 500 using DDM

Using the DDM, we estimate the cash flow to investors over the next five years, and then estimate a terminal value, which is the value of cash flows from the sixth year to eternity.

Cash flow to equity investors consists of dividends [6] and buybacks [7], hence we estimated these two components individually. In order to estimate cash flows over the next five years we used an estimate of the Earnings Per Share (EPS) growth rate, since the dividends and buybacks are expected to increase in line with the EPS.

In order to estimate the EPS growth rate, we noted that the index is comprised of ten sectors, which are expected to have different growth rates. For example, it is expected that the EPS for the utilities sector increases at a much slower rate than that for the telecommunications sector, due to the rapid rate of innovation of the latter. Thus, we estimated the EPS growth rate for each sector individually [6], and then calculated a weighted average EPS growth rate for each year, using the expected representation of each sector in the index. [8] We assumed that the representation distribution over the next five years will remain similar to the current distribution. Moreover, we utilized analysts' estimates for the EPS growth rate for years 2016 and 2017 [6], while assuming the growth rate for years 2018 to 2020 to be a simple average growth rate from year 2011 to 2017. We disregarded the EPS growth rates from year 2010, as these rates were abnormally high, due to the post-recession recovery. Additionally, the EPS of energy sector have been affected dramatically by the recent drop in the oil prices, thus we assumed that the growth rate for years 2018 to 2020 will be the average growth rate for years 2011 to 2014. We also assumed the EPS growth rate of the energy sector for 2016 to be 0%, due to the negative EPS posted for year 2015.

In order to calculate the perpetuity value of dividends and buybacks, we assumed a growth rate of 2.4%, which is the current year-on-year US GDP growth rate.[9] This allowed us the calculate the terminal value using the formula below:.

Terminal value of the index = Expected Dividends in 6th year / r-g

Next, we calculated the Net Present Values (NPV) for the cash flows over the next five years, as well as the terminal value. This involved discounting the cash flows by the cost of capital using the following formula:.

NPV = Dividend / (1 + Cost of capital)^Year

In order to estimate the cost of capital we have used the equation:

Cost of Capital = Risk-Free Rate + Risk Premium

The Risk-Free rate is the theoretical return of an investment with zero risk. It represents the return an investor can expect from an absolutely risk-free investment[1]. In order to estimate this, we have used the current return rate of a 10-year U.S. Treasury bond.

The Risk Premium is the return in excess of the risk-free rate of return that an investment in stocks is expected to yield[3]. It is essentially the extra return an investor gets for taking on extra risk by investing in stocks as opposed to government bonds. In order to estimate this, we can use the equation:

Risk Premium = Average Return Rate of S&P 500 - Average Return Rate of 10-year U.S. Treasury Bond

The averages over different periods are shown in table 1. Also, the averages calculated are geometric averages because, as argued by Aswath Damodaran, empirical studies seem to indicate that returns on stocks are negatively correlated over time. An arithmetic mean would have been appropriate if annual return were uncorrelated over time[2]. See the table below for results

Period	Duration (Years)	S&P 500 Avg Return	10-year T. Bond Avg Return	Risk Premium
1928-2015	88	9.50%	4.96%	4.54%
1966-2015	50	9.61%	6.71%	2.90%
2006-2015	10	7.25%	4.71%	2.53%

Table 1: Data collected from reference [4]

Using the 10-year US treasury bond return rate (31-12-2015) of 2.269%[5] as the risk-free rate, and the 88-year average risk premium of 4.54%, the cost of capital is therefore 6.8%. We use this value for the years 2015 to 2020. The cost of capital at 6.8% is below the historical 88-year average of 9.5%[4], therefore we use 9.5% for the terminal value.

This resulted in an intrinsic value of the S&P 500 index to be 2711.59, which suggests that the index is currently undervalued when compared to the 31st of December 2015 closing value of 2,043.94[10]. This is illustrated in Table 2, and a more detailed calculation can be found in Appendix A. The difference between our estimate and the index value could be because analysts currently anticipate a high growth phase between 2016 and 2020. If we were to set a constant growth rate for that period to 6%, we end up with an estimate of the value of the index of 2012.93; much in line with the December 2015 closing value.

	2015	2016	2017	2018	2019	2020	Terminal Year
Buyback	$67.53	$76.63	$96.06	$105.35	$115.55	$126.73	
Growth rate	4.71%	13.47%	25.36%	9.68%	9.68%	9.68%	2.40%
Dividend	$43.39	$49.23	$61.72	$67.69	$74.24	$81.42	
Dividend + Buyback	$110.92	$125.86	$157.77	$173.04	$189.78	$208.15	$3,002.02
Risk-free rate	2.27%	2.27%	2.27%	2.27%	2.27%	2.27%	2.27%
Risk premium	4.54%	4.54%	4.54%	4.54%	4.54%	4.54%	4.54%
Cost of equity	6.81%	6.81%	6.81%	6.81%	6.81%	6.81%	9.50%
NPV	$110.92	$117.83	$138.30	$142.01	$145.82	$149.74	$1,906.96
Intrinsic value	$2711.5857						

Table 2: S&P 500 valuation

Drawbacks to using the DDM

The main drawback of the dividend discount model is caused by the uncertainty involved in forecasting the future dividends paid to the shareholder. These forecasts are difficult to estimate due to the uncertain nature of forecasting a company's future earnings, and cost of capital.

Sanity Checks

Using other methodologies, we calculated the fair value of the S&P 500. The Discounted Cash Flow method yielded a fair value for the index of 1833, suggesting that the index is undervalued. We also explored determining whether the index was overvalued or not using comparable methodologies. A P/E ratio based evaluation suggested that the index is slightly overvalued (Appendix 1). Instead a Fed model based evaluation suggested that the index is largely undervalued and has been since the year 2000 (Appendix 2). However, since the latter is considered to be highly unreliable, it is reasonable to conclude the S&P 500 is currently overvalued even though the exact fair value is difficult to assess.

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Appendix 1: *Price-Earning ratio based evaluation*

An initial estimate of the S&P 500 market value fairness can be obtained by considering its P/E ratio. Now, the last reported P/E ratio was 21.18 based on the actual reported earnings on the 30th of September 2015 [14]. However, this number alone is not helpful in evaluating the fairness of S&P 500 index. Instead, it provides valuable information when compared to the average historical P/E. The following figure shows the historical evolution of the P/E ratio for the S&P 500 index.

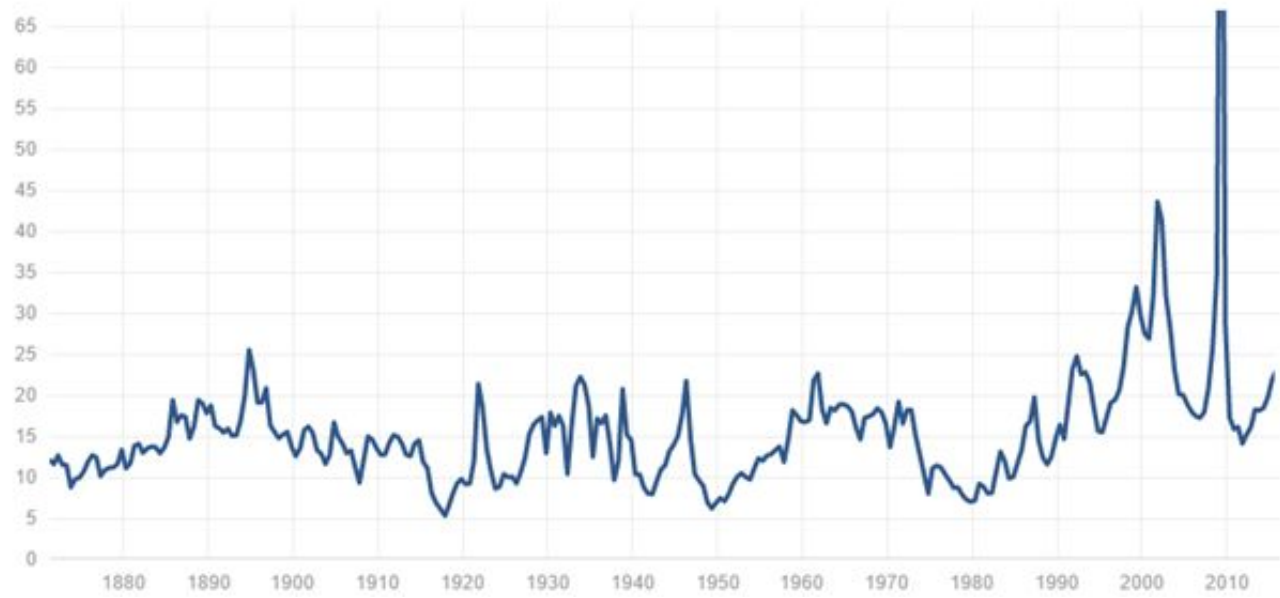

Figure A: Graph on the S&P 500 historical P/E ratio [14]

From the historical data, the mean P/E ratio is calculated to be 15.58. Therefore, since the latest reported P/E ratio of 21.18 is significantly higher than the historical average, the S&P 500 index is expected to be overvalued. In other words, the stock market prices of the individual companies do not reflect their respective earnings per share, resulting in an excessive increase in market prices. However, even though suggesting overvaluation, this substantial gap between the current and average value of the P/E ratio probably overestimates the level of overvaluation. In fact, while the global P/E average is effectively 15.58, the average since 1985 is 19.6. This suggests that the sustainable P/E increased over the years. This can be in part explained by the lower returns expected by today's investors mainly due to lower interest rates [15]. By analysing the P/E ratio, the S&P 500 index is hence confirmed to be slightly overvalued.

Appendix 2: *Fed model evaluation*

The Fed model, closely related to the DDM, can also be used to value to S&P 500 index. It suggests that the difference between the risk free rate, usually the 10-year Treasury bond yield to maturity, and the forward earnings yield of the S&P 500 index gives an indication of fair valuation, overvaluation or undervaluation.

This model is essentially represented with the following equation: $\frac{E}{P} = Y$, where $\frac{E}{P}$ is the forward earnings yield (inverse of price-earnings ratio) and Y is the risk-free rate (treasury bonds). The difference between those two parameters gives an indication of how the S&P 500 is valued, according to the following conditions:

if $\frac{E}{P} = Y$, the stock price is fairly valued.
if $\frac{E}{P} > Y$, the stock price is undervalued.
if $\frac{E}{P} < Y$, the stock price is overvalued.

The advantages of the Fed model will now be discussed. First of all, it gives a good analysis of which competing asset an investor should buy. As the model compares the yield an investor receives on an equity investment and the yield obtained with ten year treasury bonds, the investor has a clear indication suggesting him to either invest in stock shares or in treasury bonds.

The next advantage is called the PV argument [13], which essentially means the present value of the returns will diminish with the increase of interest rates. The stock price today is the discounted present value (PV) of the future cashflows to investors from the market. When interest rates increase, the PV of future cash flows decreases, hence making the price-earning ratio fall. This fits the relation between the interest rates (Y) and the earnings yield E/P given by the Fed model.

The last advantage of the Fed model is its strong empirical support. Indeed, as shown in the figure below, the S&P 500's earnings yield and the 10-year treasury bonds yield are closely related before 2000. This support is however not true anymore, since 2000.

Figure B: Graph of the S&P 500 earnings yield and nominal bond yield (1979-2015) [11]

According to this figure, since the Tech bubble burst of 2000, the Fed model suggests the S&P 500 index is largely undervalued. This is actually not a good valuation of the S&P 500 for the following reasons.

The earnings yield $\frac{E}{P}$ is a real expected return, which is here compared to a nominal rate of return Y. This is thus quite misleading as inflation is not really taken into account. The Fed model comes from the following equation $\frac{E}{P} = Y + k - Ie$ [12] where k is the equity risk premium (excess return) and Ie is the expected inflation. But the Fed model is obtained with the very restrictive and simplifying assumption of a payout rate of 100%, which implies a zero long-term growth, hence giving the simple $\frac{E}{P} = Y$ equation. The resulting conclusion obtained from the Fed model should therefore not be taken into consideration, as this one is unreliable.